Calculs divers - corrigé

Exercice 1

$$A = 2^3 \times (4 \times 3)^2 = 2^3 \times 4^2 \times 3^2 = 2^3 \times (2^2)^2 \times 3^2$$

$$= 2^3 \times 2^4 \times 3^2 = 2^7 \times 3^2$$

$$B = \frac{7^{-10}}{7^4} = 7^{-14} = \frac{1}{7^{14}}$$

$$C = \frac{(2^2)^3 \times 10 \times 4^{-2}}{15 \times 8} = \frac{2^6 \times 5 \times 2 \times (2^2)^{-2}}{5 \times 3 \times 2^3} = \frac{2^7 \times 2^{-4}}{3 \times 2^3}$$

$$= \frac{2^3}{3 \times 2^3} = \frac{1}{3}$$

$$D = \frac{5 \times 10^{-3} \times 12 \times 10^4}{15 \times 10^2 \times 8 \times 10^{-5}} = \frac{5 \times 4 \times 3 \times 10^1}{5 \times 3 \times 4 \times 2 \times 10^{-3}} = \frac{10^4}{2} = 5\,000$$

Exercice 2

$$A = \frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$B = \frac{\frac{3}{4}}{2} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

$$C = \frac{3}{\frac{4}{2}} = 3 \times \frac{2}{4} = 3 \times \frac{1}{2} = \frac{3}{2}$$

$$D = \frac{1}{2 + \frac{3}{2}} = \frac{1}{\frac{4}{2} + \frac{3}{2}} = \frac{1}{\frac{7}{2}} = \frac{2}{7}$$

$$E = \frac{4}{3} \times 9 = \frac{4 \times 9}{3} = \frac{4 \times 3 \times 3}{3} = \frac{4 \times 3}{1} = 12.$$

$$F = \frac{3}{4} \times \frac{-16}{21} = -\frac{3 \times 16}{4 \times 21} = -\frac{3 \times 4 \times 4}{4 \times 7 \times 3} = -\frac{4}{7}.$$

Exercice 3

$$A = \sqrt{9} - \sqrt{36} = \sqrt{3^2} - \sqrt{6^2} = 3 - 6 = -3$$

$$B = \frac{2}{\sqrt{2}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$C = \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

$$D = \frac{\sqrt{2}}{\sqrt{8}} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}.$$

Exercice 4

$$A = \sqrt{5} \times \sqrt{15} = \sqrt{5} \times \sqrt{5\times 3} = \sqrt{5} \times \sqrt{5} \times \sqrt{3} = 5\sqrt{3}$$

$$B = \sqrt{75} = \sqrt{25\times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

$$C = \frac{\sqrt{84}}{\sqrt{7}} = \frac{\sqrt{2\times 42}}{\sqrt{7}} = \frac{\sqrt{2\times 6\times 7}}{\sqrt{7}} = \frac{\sqrt{12}\times\sqrt{7}}{\sqrt{7}} = \sqrt{12} = \sqrt{4\times 3}$$

$$= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}.$$

$$D = (2+\sqrt{3})^2 = 2^2 + 2\times 2\times\sqrt{3} + (\sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}.$$

Exercice 5

$$A = (3x+5)(1-x) = 3x - 3x^2 + 5 - 5x = -3x^2 - 2x + 5$$

$$B = (2-3x)^2 = 2^2 - 2\times 2\times(3x) + (3x)^2 = 4 + 12x + 9x^2$$

$$C = (2x+3x^2)^2 = (2x)^2 + 2\times(2x)\times(3x^2) + (3x^2)^2$$

$$= 4x^2 + 12x^3 + 9x^4.$$

$$D = (1-2x)(1+2x) - 3x(5-x)$$

$$= 1^2 - (2x)^2 - (15x - 3x^2)$$

$$= 1 - 4x^2 - 15x + 3x^2$$

$$= -x^2 - 15x + 1$$

$$E = 2 - 4x - 3\left((1-x)^2 - (5-x)\right)$$

$$= 2 - 4x - 3\left(1 - 2x + x^2 - 5 + x\right)$$

$$= 2 - 4x - 3\left(x^2 - x - 4\right)$$

$$= 2 - 4x - 3x^2 + 3x + 12$$

$$= -3x^2 - x + 14$$

Exercice 6

$$A = 2 + \frac{3}{x+2} = \frac{2(x+2)}{x+2} + \frac{3}{x+2}$$

$$= \frac{2x+4+3}{x+2} = \frac{2x+7}{x+2}$$

$$B = \frac{2x}{1-x} - \frac{3+x}{4x} = \frac{(2x)(4x)}{4x(1-x)} - \frac{(3+x)(1-x)}{4x(1-x)}$$

$$= \frac{8x^2 - (3 - 3x + x - x^2)}{4x(1-x)}$$

$$= \frac{8x^2 - 3 + 3x - x + x^2}{4x(1-x)}$$

$$= \frac{9x^2 + 2x - 3}{4x(1-x)}$$

$$C = \frac{2}{x-3} - \frac{1}{x^2+1} = \frac{2(x^2+1)}{(x-3)(x^2+1)} - \frac{x-3}{(x^2+1)(x-3)}$$

$$= \frac{2x^2+2-(x-3)}{(x-3)(x^2+1)} = \frac{2x^2 - x + 5}{(x-3)(x^2+1)}$$

$$D = \frac{1-x^2}{(x-1)(2-x)} = \frac{(1-x)(1+x)}{(x-1)(2-x)} = \frac{-(x-1)(1+x)}{(x-1)(2-x)} = -\frac{1+x}{2-x}$$

Exercice 7

$$A = 6x^2 - x = x(6x-1)$$

$$B = 4x^2 + 4x + 1 = (2x)^2 + 2 \times (2x) \times 1 + 1^2 = (2x+1)^2$$

$$C = x^2 - 6x + 9 = x^2 - 2 \times x \times 3 + 3^2 = (x-3)^2$$

$$D = 9x^2 - 1 = (3x)^2 - 1^2 = (3x-1)(3x+1)$$

$$E = (1+x)^2 - 4 = (1+x)^2 - 2^2$$

$$= ((1+x) - 2)\left((1+x) + 2\right)$$

$$= (x-1)(x+3)$$

$$F = x^2 - 4 - (x+2)(5x+3) = (x-2)(x+2) - (x+2)(5x+3)$$

$$= (x+2)\left((x-2) - (5x+3)\right) = (x+2)(-4x-5) = -(x+2)(4x+5)$$