TD AL 1 : Systèmes linéaires

Corrigé des ADC

ADC 1

1) Par substitution

$$\begin{cases} 2x + y = 5 \\ 4x + 3y = -2 \end{cases} \iff \begin{cases} y = 5 - 2x \\ 4x + 3(5-2x) = -2 \end{cases}$$

$$\iff \begin{cases} y = 5 - 2x \\ -2x + 15 = -2 \end{cases}$$

$$\iff \begin{cases} y = 5 - 2x \\ -2x = -17 \end{cases}$$

$$\iff \begin{cases} y = -12 \\ x = \frac{17}{2} \end{cases}$$

Il y a une solution : $\left(\frac{17}{2}, -12\right)$

Vérification : $\begin{cases} 2 \times \frac{17}{2} + (-12) = 17 - 12 = 5 \\ 4 \times \frac{17}{2} + 3 \times (-12) = 2 \times 17 + 3 \times (-12) = 34 - 36 = -2 \end{cases}$

Par pivot de Gauss

$$\begin{cases} 2x + y = 5 \\ 4x + 3y = -2 \end{cases} \iff \begin{cases} 2x + y = 5 \\ -2x = -17 \end{cases} \quad L_2 \leftarrow L_2 - 3L_1$$

$$\iff \begin{cases} 2x + y = 5 \\ x = \frac{17}{2} \end{cases}$$

$$\iff \begin{cases} y = -12 \\ x = \frac{17}{2} \end{cases}$$

2) $$\begin{cases} 2x + 4y = y \\ 5x + 6y = x \end{cases} \iff \begin{cases} 2x + 3y = 0 \\ 4x + 6y = 0 \end{cases}$$

⚠ On passe toutes les inconnues à gauche avant de commencer.

$$\iff 2x + 3y = 0 \quad \text{car } L_2 = 2L_1$$

$$\iff x = -\frac{3}{2}y$$

$$\iff (x, y) = \left(-\frac{3}{2}y, y\right)$$

L'ensemble des solutions est $\left\{\left(-\frac{3}{2}y, y\right) \text{ avec } y \in \mathbb{R}\right\}$.
Il est infini.

3) $\begin{cases} 3x - y = 1 \\ 6x - 2y = 3 \end{cases} \iff \begin{cases} y = 3x - 1 \\ 6x - 2(3x-1) = 3 \end{cases}$

$\iff \begin{cases} y = 3x - 1 \\ 2 = 3 \end{cases}$ impossible.

Il n'y a pas de solution.

ADC2

1) $\begin{cases} x + y + z = 1 \\ 2y + 2z = 1 \\ 4z = 1 \end{cases}$ système triangulaire

$\iff \begin{cases} x+y = 1 - \frac{1}{4} \\ 2y = 1 - \frac{1}{2} \\ z = \frac{1}{4} \end{cases} \iff \begin{cases} x = \frac{3}{4} - \frac{1}{4} \\ y = \frac{1}{4} \\ z = \frac{1}{4} \end{cases}$

$\iff \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{4} \\ z = \frac{1}{4} \end{cases}$

Il y a une seule solution : $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$.

2) $\begin{cases} x + y + z = 1 \\ 3x - 2y + 2z = 1 \\ 4z = -2 \end{cases} \iff \begin{cases} x+y = 1 + 1/2 \\ 3x - 2y = 1 + 1 \\ z = -\frac{1}{2} \end{cases}$

$\iff \begin{cases} x + y = 3/2 \\ 3x - 2y = 2 \\ z = -1/2 \end{cases}$ (S)

Méthode 1 : substitution

(S) $\iff \begin{cases} x = \frac{3}{2} - y \\ 3\left(\frac{3}{2} - y\right) - 2y = 2 \\ z = -1/2 \end{cases} \iff \begin{cases} x = 3/2 - y \\ -5y = -\frac{5}{2} \\ z = -\frac{1}{2} \end{cases}$

$\iff \begin{cases} x = 1 \\ y = 1/2 \\ z = -1/2 \end{cases}$

Il y a une seule solution : $\left(1, \frac{1}{2}, -\frac{1}{2}\right)$

Méthode 2 : par pivot :

$$(S) \iff \begin{cases} x + y = 3/2 \\ -5y = -\frac{5}{2} \\ z = -1/2 \end{cases} \qquad L_2 \leftarrow L_2 - 3L_1$$

$$\iff \begin{cases} x = 1 \\ y = \frac{1}{2} \\ z = -\frac{1}{2} \end{cases}$$

ADC 3

1) $$\begin{cases} x - y + 2z = 0 \\ x + y - z = 0 \\ x - 5y + 8z = 0 \end{cases} \iff \begin{cases} x - y + 2z = 0 \\ 2y - 3z = 0 \\ -4y + 6z = 0 \end{cases} \qquad \begin{matrix} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \end{matrix}$$

$$\iff \begin{cases} x + \frac{1}{2}z = 0 \\ 2y - 3z = 0 \\ 0 = 0 \end{cases} \qquad \begin{matrix} L_1 \leftarrow L_1 + \frac{1}{2}L_2 \\ L_3 \leftarrow L_3 + 2L_2 \end{matrix}$$

$$\iff \begin{cases} x = -\frac{1}{2}z \\ y = \frac{3}{2}z \end{cases} \qquad z \text{ quelconque}$$

$$\iff (x, y, z) = \left(-\frac{1}{2}z ; \frac{3}{2}z ; z\right)$$

L'ensemble des solutions est :

$$\left\{ \left(-\frac{1}{2}z , \frac{3}{2}z , z\right) , z \in \mathbb{R} \right\}$$

2) $\begin{cases} x - y + 2z = 2 \\ x + 3y - z = 1 \\ x - 5y + 6z = 6 \end{cases} \iff \begin{cases} x - y + 2z = 2 \\ 4y - 3z = -1 \\ -4y + 4z = 4 \end{cases}$ $\quad L_2 \leftarrow L_2 - L_1$, $L_3 \leftarrow L_3 - L_1$

$\iff \begin{cases} x - y + 2z = 2 \\ 4y - 3z = -1 \\ -y + z = 1 \end{cases}$ $\quad L_3 \leftarrow \frac{1}{4} L_3$

$\iff \begin{cases} x + z = 1 \\ z = 3 \\ -y + z = 1 \end{cases}$ $\quad L_1 \leftarrow L_1 - L_3$, $L_2 \leftarrow L_2 + 4L_3$

$\iff \begin{cases} x = -2 \\ z = 3 \\ y = 2 \end{cases}$

Une seule solution : $(-2, 2, 3)$.

pivot facile → rien à faire

3) $\begin{cases} 2x - y - z + t = -1 \\ 3x - 3y = 9 \\ x - z = 6 \\ x - y - z = 2 \end{cases}$

pivot 2

$\iff \begin{cases} x - y + t = -7 \\ 3x - 3y = 9 \\ x - z = 6 \\ -y = -4 \end{cases}$ $\quad L_1 \leftarrow L_1 - L_3$, $L_4 \leftarrow L_4 - L_3$

pivot 3

$\iff \begin{cases} x + t = -3 \\ 3x = 21 \\ x - z = 6 \\ y = 4 \end{cases}$ $\quad L_1 \leftarrow L_1 - L_3$, $L_2 \leftarrow L_2 - 3L_3$

$\iff \begin{cases} t = -10 \\ x = 7 \\ z = 1 \\ y = 4 \end{cases}$

Une seule solution : $(7, 4, 1, -10)$

4) $\begin{cases} x - 2y + z + 3t = 0 \\ 2x + y - z + 5t = 0 \end{cases}$ $\iff \begin{cases} x - 2y + z + 3t = 0 \\ 5y - 3z - t = 0 \end{cases}$ $L_2 \leftarrow L_2 - 2L_1$

$\iff \begin{cases} x + 13y - 8z = 0 \\ 5y - 3z - t = 0 \end{cases}$ $L_1 \leftarrow L_1 + 3L_2$

$\iff \begin{cases} x = -13y + 8z \\ t = 5y - 3z \end{cases}$ y, z quelconques

$\iff (x, y, z, t) = (-13y + 8z, y, z, 5y - 3z)$

L'ensemble des solutions est :

$$\{(-13y + 8z, y, z, 5y - 3z) \text{ avec } (y, z) \in \mathbb{R}^2\}$$