TD AL1 – Corrigé des ADC

ADC 1

- $z_1 = (3+2i)^2\,(2-i) = (9+12i+4i^2)(2-i)$

$= (5+12i)(2-i) = 22+19i$

Donc $Re(z_1) = 22$ et $Im(z_1) = 19$

- $z_2 = \dfrac{(3+2i)(1+i)}{1-i} = \dfrac{(3+2i)(1+i)^2}{(1-i)(1+i)} = \dfrac{(3+2i)\times 2i}{2} = 3i-2$

Donc $Re(z_2) = -2$ et $Im(z_2) = 3$

ADC 2

$|z_3| = \dfrac{|1+i|}{|1-3i|} = \dfrac{\sqrt{2}}{\sqrt{10}} = \dfrac{\sqrt{5}}{5}$.

$|z_4| = |2+3i|^4 = 13^2 = 169$

ADC 3

- $$\cos(2x)\sin^3(x) = \left(\frac{e^{2ix}+e^{-2ix}}{2}\right)\left(\frac{e^{ix}-e^{-ix}}{2i}\right)^3$$

$$= -\frac{1}{16i}\left(e^{2ix}+e^{-2ix}\right)\left(e^{3ix}-3e^{ix}+3e^{-ix}-e^{-3ix}\right)$$

$$= -\frac{1}{16i}\left(e^{5ix}-3e^{3ix}+3e^{ix}-e^{-ix}+e^{ix}-3e^{-ix}+3e^{-3ix}-e^{-5ix}\right)$$

$$= -\frac{1}{16i}\left(\left(e^{5ix}-e^{-5ix}\right)-3\left(e^{3ix}-e^{-3ix}\right)+4\left(e^{ix}-e^{-ix}\right)\right)$$

$$= -\frac{1}{16i}\left(2i\sin(5x)-6i\sin(3x)+8i\sin(x)\right)$$

$$= -\frac{1}{8}\sin(5x)+\frac{3}{8}\sin(3x)-\frac{1}{2}\sin(x)$$

- $$\sin^4(x) = \left(\frac{e^{ix}-e^{-ix}}{2i}\right)^4 = \frac{1}{16}\left(e^{2ix}-2+e^{-2ix}\right)\left(e^{2ix}-2+e^{-2ix}\right)$$

$$= \frac{1}{16}\left(e^{4ix}+e^{-4ix}-4\left(e^{2ix}+e^{-2ix}\right)+6\right)$$

$$= \frac{1}{8}\cos(4x)-\frac{1}{2}\cos(2x)+\frac{3}{4}.$$

ADC4

- $z_5 = 57\, e^{i \times 0}$
- $z_6 = 4\, e^{-i\pi/2}$
- $z_7 = 2\, e^{2i\pi/3}$
- $z_8 = 2\sqrt{2}\, e^{-i\pi/4} = 2\sqrt{2}\, e^{7i\pi/4}$
- $z_9 = \dfrac{2e^{2i\pi/3}}{2\sqrt{2}\, e^{-i\pi/4}} = \dfrac{\sqrt{2}}{2}\, e^{11i\pi/12}$
- $z_{10} = 2e^{2i\pi/3} \times 2\sqrt{2}\, e^{-i\pi/4} = 4\sqrt{2}\, e^{5i\pi/12}$

ADC5

$$z_{11} = \left(\frac{2e^{-i\pi/6}}{\sqrt{2}\, e^{-i\pi/4}} \right)^{10} = (\sqrt{2})^{10} \left(e^{i\pi/12} \right)^{10} = 32\, e^{5i\pi/6}$$

$$= 32\left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i \right) = -16\sqrt{3} + 16i$$

ADC6

Soit $z \in \mathbb{C}$.

* $z = 0$ n'est pas solution de $z^2 = -1 + i\sqrt{3}$

* Prenons $z = re^{i\theta} \in \mathbb{C}^*$ avec $r > 0$, $\theta \in \mathbb{R}$.

$$z^2 = -1 + i\sqrt{3} \iff r^2 e^{2i\theta} = 2\, e^{2i\pi/3}$$

$$\iff \begin{cases} r^2 = 2 \\ \exists k \in \mathbb{Z},\ 2\theta = \frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\iff \begin{cases} r = \sqrt{2} & (\text{car } r > 0) \\ \exists k \in \mathbb{Z},\ \theta = \frac{\pi}{3} + k\pi \end{cases}$$

$$\iff \exists k \in \mathbb{Z},\ z = \sqrt{2}\, e^{i(\pi/3 + k\pi)}$$

Il y a en fait deux solutions : $z_0 = \sqrt{2} e^{i\pi/3}$ et $z_1 = \sqrt{2}\, e^{4i\pi/3} = -z_0$