TD AL3 : Corrigé des ADC

ADC 1

$$P+Q = (X^2+3X-2)+(6X-X^2+1) = 9X-1$$

$$3P-2Q = 3(X^2+3X-2)-2(6X-X^2+1) = 5X^2-3X-8$$

$$P^3 = (X^2+3X-2)(X^2+3X-2)(X^2+3X-2)$$

$$= (X^4+6X^3+5X^2-12X+4)(X^2+3X-2)$$

$$= X^6+9X^5+21X^4-9X^3-42X^2+36X-8$$

$$PQ = (X^2+3X-2)(-X^2+6X+1)$$

$$= -X^4+3X^3+21X^2-9X-2$$

$$P(Q(X)) = Q(X)^2+3Q(X)-2$$

$$= (-X^2+6X+1)^2+3(-X^2+6X+1)-2$$

$$= X^4-12X^3+34X^2+12X+1 \quad -3X^2+18X+1$$

$$= X^4-12X^3+31X^2+30X+2$$

ADC 2

1) $P_1 = X^3-X^2+4X-4$ donc $\deg(P_1)=3$

2) $P_2 = X^3-X(X^2-4X+4) = 4X^2-4X$ donc $\deg(P_2)=2$

3) $P_3 = (X+1)^{10}-(4X^2+1)^8$.

$\rightarrow \deg((X+1)^{10}) = 10\times\deg(X+1) = 10\times 1 = 10$

$\rightarrow \deg((4X^2+1)^8) = 8\times\deg(4X^2+1) = 8\times 2 = 16 > \deg((X+1)^{10})$

Donc $\deg(P_3) = \deg((4X^2+1)^8) = 16$.

4) $Q_n = \prod_{k=1}^{n}(2X^k-1)$

$$\deg(Q_n) = \sum_{k=1}^{n}\deg(2X^k-1) = \sum_{k=1}^{n}k = \frac{n(n+1)}{2}$$

ADC 3

1) Soit $P \in \mathbb{R}_1[X]$. $P = aX + b$ avec $a, b \in \mathbb{R}$

$$\begin{cases} P(-1) = -2 \\ P(0) = 1 \end{cases} \iff \begin{cases} -a + b = -2 \\ b = 1 \end{cases} \iff \begin{cases} a = 3 \\ b = 1 \end{cases}$$

L'ensemble des solutions est $\{3X+1\}$.

2) Soit $P \in \mathbb{R}_2[X]$, $P = aX^2 + bX + c$ avec $a, b, c \in \mathbb{R}$

$$\begin{cases} P(-1) = -2 \\ P(0) = 1 \\ P(1) = 0 \end{cases} \iff \begin{cases} a - b + c = -2 \\ c = 1 \\ a + b + c = 0 \end{cases} \iff \begin{cases} a - b = -3 \\ c = 1 \\ a + b = -1 \end{cases}$$

$$\iff \begin{cases} a = b - 3 \\ c = 1 \\ 2b - 3 = -1 \end{cases} \iff \begin{cases} a = -2 \\ c = 1 \\ b = 1 \end{cases}$$

L'ensemble des solutions est $\{-2X^2 + X + 1\}$

ADC 4

1)

$$\begin{array}{r|l} X^3 + 1 & X^2 + X + 1 \\ \cline{2-2} -(X^3 + X^2 + X) & X - 1 \\ \cline{1-1} -X^2 - X + 1 & \\ -(-X^2 - X - 1) & \\ \cline{1-1} 2 & \end{array}$$

reste : 2 $\quad \deg(2) < \deg(X^2+X+1)$

quotient : $X - 1$

$X^3 + 1 = (X-1)(X^2+X+1) + 2$

2)

$$\begin{array}{r|l} 2X^5 + X^3 + 17X - 2 & X^2 + 2X + 3 \\ \cline{2-2} -(2X^5 + 4X^4 + 6X^3) & 2X^3 - 4X^2 + 3X + 6 \\ \cline{1-1} -4X^4 - 5X^3 + 17X - 2 & \\ -(-4X^4 - 8X^3 - 12X^2) & \\ \cline{1-1} 3X^3 + 12X^2 + 17X - 2 & \\ -(3X^3 + 6X^2 + 9X) & \\ \cline{1-1} 6X^2 + 8X - 2 & \\ -(6X^2 + 12X + 18) & \\ \cline{1-1} -4X - 20 & \end{array}$$

$\deg(-4X - 20) < \deg(X^2 + 2X + 3)$

$2X^5 + X^3 + 17X - 2 = (X^2 + 2X + 3)(2X^3 - 4X^2 + 3X + 6) - 4X - 20$

ADC 5

$$P = -3X^2 + 6X - 6 .$$

$$\Delta(P) = 6^2 - 4 \times (-3) \times (-6) = 6^2 - 12 \times 6 = 6^2 - 2 \times 6^2 = -6^2$$

Les racines de P dans $\mathbb{C}$ sont :

$$z_1 = \frac{6 - 6i}{-6} = -1 + i \qquad \text{et} \qquad z_2 = -1 - i$$

$$P = -3\left(X - (-1+i) \right) \left(X - (-1-i) \right)$$

ADC 6

$$P = X^3 - X^2 - 14X + 24$$

* $P(2) = 8 - 4 - 28 + 24 = 0$ donc 2 est racine de P.

* $X - 2$ divise P.

$$
\begin{array}{r|l}
X^3 - X^2 - 14X + 24 & X - 2 \\
\cline{2-2}
-(X^3 - 2X^2) \qquad\qquad\quad & X^2 + X - 12 \\
\cline{1-1}
X^2 - 14X + 24 & \\
-(X^2 - 2X) \qquad\quad & \\
\cline{1-1}
-12X + 24 & \\
-(-12X + 24) & \\
\cline{1-1}
0 &
\end{array}
$$

Donc $\quad P = (X-2)(X^2 + X - 12)$

$$\Delta(X^2 + X - 12) = 1 - 4 \times (-12) = 49 = 7^2$$

Les racines de $X^2 + X - 12$ sont : $\dfrac{-1-7}{2} = -4$ et $\dfrac{-1+7}{2} = 3$

Donc $\quad P = (X-2)(X+4)(X-3)$