TD AL3 : Corrigé des ADC

ADC 1

$$P + Q = (X^2 + 3X - 2) + (6X - X^2 + 1) = 9X - 1$$

$$3P - 2Q = 3(X^2 + 3X - 2) - 2(6X - X^2 + 1) = 5X^2 - 3X - 8$$

$$P^3 = (X^2 + 3X - 2)(X^2 + 3X - 2)(X^2 + 3X - 2)$$

$$= (X^4 + 6X^3 + 5X^2 - 12X + 4)(X^2 + 3X - 2)$$

$$= X^6 + 9X^5 + 21X^4 - 9X^3 - 42X^2 + 36X - 8$$

$$PQ = (X^2 + 3X - 2)(-X^2 + 6X + 1)$$

$$= -X^4 + 3X^3 + 21X^2 - 9X - 2$$

$$P(Q(X)) = Q(X)^2 + 3Q(X) - 2$$

$$= (-X^2 + 6X + 1)^2 + 3(-X^2 + 6X + 1) - 2$$

$$= X^4 - 12X^3 + 34X^2 + 12X + 1 \quad - 3X^2 + 18X + 1$$

$$= X^4 - 12X^3 + 31X^2 + 30X + 2$$

ADC 2

1) $P_1 = X^3 - X^2 + 4X - 4$ donc $\deg(P_1) = 3$

2) $P_2 = X^3 - X(X^2 - 4X + 4) = 4X^2 - 4X$ donc $\deg(P_2) = 2$

3) $P_3 = (X+1)^{10} - (4X^2 + 1)^8$.

$\rightarrow \deg((X+1)^{10}) = 10 \times \deg(X+1) = 10 \times 1 = 10$

$\rightarrow \deg((4X^2+1)^8) = 8 \times \deg(4X^2+1) = 8 \times 2 = 16 > \deg((X+1)^{10})$

Donc $\deg(P_3) = \deg((4X^2+1)^8) = 16$.

ADC 3

1)

$$\begin{array}{r|l} X^3 + 1 & X^2 + X + 1 \\ \cline{2-2} -(x^3 + x^2 + x) & X - 1 \\ \cline{1-1} -x^2 - X + 1 & \\ -(-x^2 - x - 1) & \\ \cline{1-1} 2 & \end{array}$$

reste : 2 $\quad deg(2) < deg(x^2+X+1)$

quotient : $X - 1$

$X^3 + 1 = (X-1)(X^2+X+1) + 2$

2)

$$\begin{array}{r|l} 2X^5 + X^3 + 17X - 2 & X^2 + 2X + 3 \\ \cline{2-2} -(2X^5 + 4X^4 + 6X^3) & 2X^3 - 4X^2 + 3X + 6 \\ \cline{1-1} -4X^4 - 5X^3 + 17X - 2 & \\ -(-4X^4 - 8X^3 - 12X^2) & \\ \cline{1-1} 3X^3 + 12X^2 + 17X - 2 & \\ -(3X^3 + 6X^2 + 9X) & \\ \cline{1-1} 6X^2 + 8X - 2 & \\ -(6X^2 + 12X + 18) & \\ \cline{1-1} -4X - 20 & \end{array}$$

$deg(-4X - 20) < deg(X^2 + 2X + 3)$

$2X^5 + X^3 + 17X - 2 = (X^2 + 2X + 3)(2X^3 - 4X^2 + 3X + 6) - 4X - 20$

ADC 4

$P(X) = -4X^2 + 6X + 4$

$\Delta = 6^2 - 4 \times (-4) \times 4 = 100 > 0$ donc P admet deux racines réelles distinctes :

$$x_1 = \frac{-6 - \sqrt{100}}{2 \times (-4)} = \frac{-6-10}{-8} = 2$$

$$x_2 = \frac{-6 + \sqrt{100}}{2 \times (-4)} = \frac{-6+10}{-8} = -\frac{1}{2}$$

Donc

$$P(X) = -4\left(X - 2\right)\left(X + \frac{1}{2}\right)$$

ADC 5

$$P = X^3 - X^2 - 14X + 24$$

* $P(2) = 8 - 4 - 28 + 24 = 0$ donc 2 est racine de P.

* $X-2$ divise P.

$$\begin{array}{r|l} X^3 - X^2 - 14X + 24 & X-2 \\ \cline{2-2} -(X^3 - 2X^2) & X^2 + X - 12 \\ \cline{1-1} X^2 - 14X + 24 & \\ -(X^2 - 2X) & \\ \cline{1-1} -12X + 24 & \\ -(-12X + 24) & \\ \cline{1-1} 0 & \end{array}$$

Donc $P = (X-2)(X^2 + X - 12)$

$\Delta(X^2 + X - 12) = 1 - 4 \times (-12) = 49 = 7^2$

Les racines de $X^2 + X - 12$ sont : $\frac{-1-7}{2} = -4$ et $\frac{-1+7}{2} = 3$

Donc $P = (X-2)(X+4)(X-3)$

ADC 6

$\begin{cases} a+b = -2 \\ ab = -15 \end{cases} \iff$ a et b sont les racines de

$$P(X) = X^2 - (-2)X + (-15)$$

On a : $P(X) = X^2 + 2X - 15$

$\Delta = 4 + 4 \times 15 = 64 > 0$. donc P a deux racines :

$x_1 = \frac{-2-8}{2} = -5$ et $x_2 = \frac{-2+8}{2} = 3$

Donc $(a,b) = (-5, 3)$ ou $(a,b) = (3, -5)$